

RESISTANCE ANOMALIES IN FLOW OF POLYMER SOLUTIONS THROUGH CHANNELS OF VARIABLE SECTION

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An experimental study is performed of development of elastic turbulence and additional resistance to flows of polymer solutions in channels with periodically varying cross section.

The study of hydromechanics of polymer solutions has recently become widespread. Many of the effects observed are of great practical significance. One such effect is the significant increase in resistance which develops upon filtration of polymer solutions. The increase in resistance upon filtration of water with polymer additives is used in the petroleum extraction industry to improve petroleum output from strata using contour dehydration [1, 2].

Experimental studies of polymer liquid filtration anomalies reveal that at low velocities their behavior does not differ from that of Newtonian fluids. Then at some filtration velocity significant increases in resistance appear. The resistance may exceed the Newtonian value by 1-2 orders of magnitude [3]. The Reynolds number does not determine the moment of onset of filtration anomalies. The thinner the pores, the lower the Reynolds number at which the resistance increase appears.

It will be convenient to study the causes and principles of polymer solution filtration anomalies using a model flow of solution through variable section channels [4, 5]. Anomalies found in variable section channels preserve all the characteristic features of filtration anomalies [5]. The present study will examine the development of anomalies in flow of polymer solutions through channels with periodically varying section.

Channels were constructed with a regularly varying square cross section (Fig. 1). The channels consisted of 10 compression-expansion sections. Narrow sections with side length a and length $2.5a$ alternated with wide sections having a side length of $2.5a$ and the same length. The individual channel sections were prepared of Plexiglas on a milling machine then glued together. Flow in channels with dimension a from 0.42 to 8.4 mm was studied.

Solution flow commenced from a reference volume with variable head height. Piezometers were used to measure the pressure drop between input and output of the channel. Volume flow rate was determined by the volume method. After exit from the channel solutions were not reused, in order to eliminate effects of degradation on measurement results.

To clarify the causes responsible for development of resistance anomalies in the variable section channels, experiments were performed with flow visualization using a colored jet as well as experiments with a DISA 55L laser Doppler anemometer (Denmark).

Aqueous solutions of polyoxyethylene with a wide concentration range were used. Special attention was given to experiments with highly concentrated solutions for which methods of determining rheological characteristics are known, inasmuch as use of such solutions allows determination of quantitative principles.

The rheological characteristics of the concentrated solutions were determined by the harmonic oscillation method and in the single-direction regime using an R.18 Weissenberg rheogoniometer made by Sangamo Controls Ltd (Great Britain) [6]. A cone-plane pair were used for the working sections, the cones being 5 and 10 cm in diameter with aperture angle of 2° .

The amplitude of the harmonically varying deformation was chosen sufficiently small that the liquid behavior followed the linear rheological equation. Linear behavior was confirmed by constancy of the measured elastic characteristics upon change in oscillation amplitude.

The single-direction rotation regime was used to determine the dependence of effective viscosity on shear rate, and moreover, for some concentrated solutions measurements of normal stresses at various shear rates were performed.

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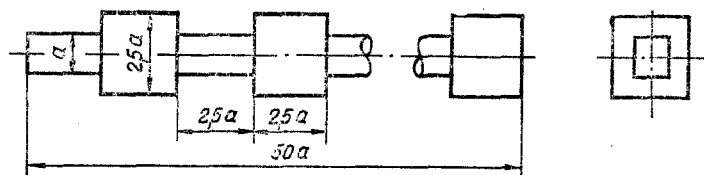


Fig. 1. Example of variable section channel.

The experiments used aqueous solutions of polyoxyethylenes polymerized to various degrees, with molecular masses exceeding $5 \cdot 10^5$. Polymer concentration in the concentrated solutions lay in the range 0.1-1%. To eliminate formation of gelatinous clumps and speed preparation of these solutions, the dry polymer powder was first dispersed in alcohol. The suspension of polymer powder in alcohol was mixed with water and maintained, with mixing from time to time, for more than a week. The quantity of alcohol added and polymer concentration were not closely controlled.

After complete solution of the polymer the concentrated solution thus prepared was used in the experiments with variable cross section channels and its rheological characteristics were determined with the rheogoniometer. Both these processes were performed at the same temperature.

In preparing the dilute solution, the polymer powders were not predispersed in alcohol. An intermediate concentration solution was prepared then diluted to the required level.

The experiments revealed that flow of Newtonian liquids in the channels studied remained laminar up to $Re = 700$. For the laminar regime the expression $\lambda = 47/Re$ is valid, while in the turbulent regime at $Re > 700$ the value of λ becomes higher.

At low Reynolds numbers the channel resistance coefficient for polymer solutions also corresponds to the relationship $\lambda = 47/Re$. In this case for the concentrated solutions the value used for viscosity η in the expression for Re was the effective viscosity corresponding to the mean shear velocity on the wall of the narrow section $8v/d$.

The experiments revealed that for any of the channels studied, even the largest, by increasing polymer concentration it is possible to achieve a situation in which the increase in the resistance coefficient above the value $\lambda = 47/Re$ occurs at values of Re less than the Newtonian critical value $Re^* = 700$. Other conditions being equal this threshold value is smaller, the smaller the characteristic channel dimension. The same tendency in change of the threshold Re is observed for increase in polymer concentration, increase in polymer molecular mass, and decrease in solution temperature. For small channels and sufficiently high solution concentration it is possible to observe development of resistance anomalies at $Re \sim 1$. At times the anomalous resistance exceeds the Newtonian value by an order of magnitude. Figure 2 shows several of the resistance curves obtained.

The flow visualization and laser Doppler anemometer experiments revealed that the appearance of resistance anomalies is related to loss of stability and development of turbulence. Figure 3 shows the behavior of a dye jet within a laminar flow of polymer solution where $\lambda = 47/Re$ and in a turbulent flow where resistance anomalies appear.

The results obtained indicate that the major cause of anomalies in filtration of polymer solution is also instability and development of turbulence.

The turbulence which occurs is so-called elastic turbulence, observed long ago in polymer processing [7], and related to the elastic properties of polymer liquids.

Analysis of the experimental data indicates that for one and the same solution at a constant temperature elastic turbulence and the related resistance anomaly in a variable section channel develop at one and the same characteristic shear velocity. In particular, the data of Fig. 2 lead to such a conclusion. The quantity v/d , proportional to the mean shear velocity on the wall in the narrow section and related to the threshold Reynolds number by the expression $v/d = \pi/4 \eta Re / \rho a^2$ remains constant for one and the same solution in channels of different size. This indicates that the parameter responsible for the appearance of elastic turbulence in variable section channels is the product of the quantity v/d multiplied by some constant with the dimensions of time, which characterizes the polymer solution. Experiments with concentrated solutions made it possible to clarify the meaning of this parameter and find its critical value for the channels studied. We will consider the results of these experiments.

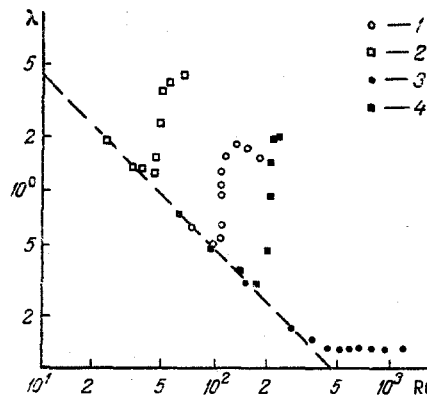


Fig. 2. Resistance of variable section channels with $\alpha = 0.58$ mm (1, 2) and $\alpha = 0.97$ mm (3, 4) for aqueous polyoxyethylene solutions with molecular mass $6.7 \cdot 10^6$ and concentrations $c = 2.5 \cdot 10^{-5}$ (1, 3) and $c = 2 \cdot 10^{-4}$ (2, 4) at 24°C . Dashed line $\lambda = 47/\text{Re}$. λ , Re dimensionless.

The ratio of the stress amplitude σ_0 to the deformation amplitude ε_0 , measured by the forced harmonic oscillation method and the phase deformation between the deformation and stress φ for concentrated solutions at various frequencies were used to determine the elastic modulus G' , the loss modulus G'' [8], and the relaxation time spectrum t_F introduced by Weissenberg [6]. These quantities were calculated with the expressions

$$G'(\omega) = \frac{\sigma_0}{2\varepsilon_0} \cos \varphi, \quad G''(\omega) = \frac{\sigma_0}{2\varepsilon_0} \sin \varphi, \quad t_F(\omega) = \frac{\text{ctg } \varphi}{\omega}. \quad (1)$$

As an example, Fig. 4 shows measurement results for one of the concentrated solutions.

Over the frequency range studied, the data obtained by the harmonic oscillation method can be described well by the Jeffries model [9]

$$\left(1 + T_1 \frac{\partial}{\partial t}\right) \sigma_{ij} = 2\eta_0 \left(1 + T_2 \frac{\partial}{\partial t}\right) \dot{\varepsilon}_{ij}. \quad (2)$$

It can easily be shown that for a viscoelastic liquid described by Jeffries equation (2), the equations

$$G'(\omega) = \frac{\eta_0(T_1 - T_2)\omega^2}{1 + T_1^2\omega^2}, \quad G''(\omega) = \frac{\eta_0(1 + T_1T_2\omega^2)\omega}{1 + T_1^2\omega^2}, \quad (3)$$

$$t_F(\omega) = \frac{T_1 - T_2}{1 + T_1T_2\omega^2}$$

are valid. In accordance with these expressions rheological constants were selected to best satisfy the data obtained experimentally by the oscillation and single-direction rotation methods. The rheological characteristics determined for all nine concentrated solutions studied are presented in Table 1. The data of Fig. 4 correspond to solution number 5. The solid lines of Fig. 4 indicate values of G' , G'' , and t_F calculated with Eq. (3) using the constants presented in Table 1.

Table 1 also shows values of θ , the elastic stress relaxation time for low perturbation frequencies, or in the terminology introduced in Truesdell's studies [10, 11], the natural time of the liquid:

$$\theta = \lim_{\omega \rightarrow 0} t_F(\omega). \quad (4)$$

For the Jeffries model, in accordance with Eq. (3), $\theta = T_1 - T_2$.

The remaining columns of Table 1 present results obtained in the experiments with variable section channels. Channel dimensions, Reynolds number, values of the dimensionless complexes T_1v/d , T_2v/d , and Weissenberg numbers [11]

$$\text{We} = \frac{\theta v}{d} \quad (5)$$

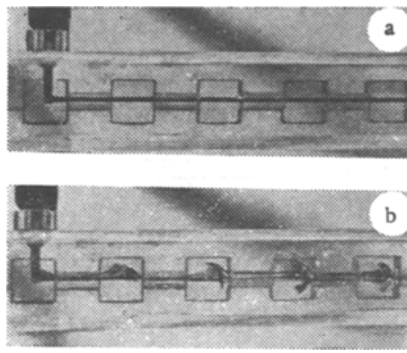


Fig. 3. Example of visualization of polymer solution flow in variable section channel: a) laminar regime with resistance anomalies absent; b) non-steady-state turbulent regime with anomalies present.

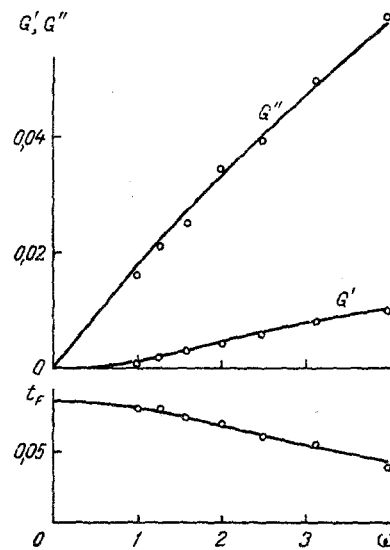


Fig. 4. Frequency dependence of rheological characteristics of one of the concentrated polymer solutions. G' and G'' , Pa; t_F , sec; ω , rad/sec.

are presented.

All dimensionless parameters were calculated from flow characteristics at the moment of stability loss and appearance of the resistance anomaly.

As follows from the data presented, elastic turbulence in the channels appears at a specific value of the Weissenberg number which does not depend on other parameters, in particular, the Reynolds number, if $Re < 440$. The critical value of the Weissenberg number comprises $We^* = 4.7$.

As is well known [11], if it is sufficiently small the Weissenberg number in a flow with constant shear is equal to the relative difference of the normal stresses:

$$We = \frac{\sigma_{11} - \sigma_{22}}{2\sigma_{12}}, \text{ if } We \ll 1. \quad (6)$$

Some authors take as the definition of the Weissenberg number not Eq. (5), but the relative difference of the normal stresses not only at small, but also at large values. Experiments were performed with some of the concentrated solutions, using the rheogoniometer to determine the quantity $(\sigma_{11} - \sigma_{22})/2\sigma_{12}$ as a function of shear velocity. Attempts to use this value as the parameter defining instability at the shear velocity corresponding to appearance of elastic turbulence in the variable section channels were unsuccessful. For the various solutions appearance of flow instability was observed at different values of the

TABLE 1. Rheological Characteristics of Solutions and Dimensionless Parameter Values for Appearance of Resistance Anomalies in Channel

Solution No.	T_1, sec	T_2, sec	$\eta_0, \text{Pa} \cdot \text{sec}$	θ, sec	a, mm	Re	$\frac{T_1 v}{d}$	$\frac{T_2 v}{d}$	We
1	0,082	0,012	0,01	0,070	4,0	190	5,39	0,79	4,60
2	0,300	0,135	0,043	0,165	4,0	36,8	9,27	4,17	5,10
3	0,370	0,270	0,055	0,100	4,0	46,8	18,3	13,4	4,95
4	0,248	0,048	0,045	0,200	4,0	23,4	5,73	1,11	4,62
5	0,290	0,205	0,018	0,085	8,4	439	15,5	11,0	4,54
6	1,20	0,52	0,16	0,68	8,4	15,2	8,47	3,67	4,80
7	0,99	0,75	0,11	0,24	8,4	25,3	18,9	14,3	4,57
8	0,76	0,29	0,06	0,47	8,4	38,2	7,40	2,82	4,58
9	0,226	0,130	0,015	0,093	8,4	348	11,3	6,53	4,67

relative normal stress difference. This is additional evidence in favor of choosing the Weissenberg parameter in the form of Eq. (5) and the dominant role of the natural time θ in analysis of viscoelastic liquid flows.

The data obtained in the experiments allow us to propose a new method for measurement of the natural time of viscoelastic liquids. The essence of the method is determination of the threshold velocity at which resistance anomalies connected with elastic turbulence develop in one or the other variable section channel. Knowing the characteristic geometric dimension and the critical value of the Weissenberg number, determined for flow of a liquid with a known natural time, the measured threshold velocity can be used to find the natural time of the test liquid $\theta = We \cdot d / v$.

As compared to other known methods the proposed one significantly decreases the lower limit of liquid viscosity and elasticity values at which natural time measurements can be performed.

As the experiments demonstrated, for aqueous solutions of polyoxyethylene with molecular mass in the millions this method can be used to measure natural time beginning at solution concentrations of several parts per million.

NOTATION

a , side length of narrow channel section; $d = 2a/\sqrt{\pi}$, hydraulic diameter of narrow channel section; l , channel length; v , mean velocity in narrow channel section; Δp , pressure head; ρ , density; $\lambda = 2d\Delta p/l\rho v^2$, hydraulic resistance coefficient; c , mass concentration; η , viscosity; η_0 , first Newtonian viscosity; t , time; ω , frequency; σ_{ij} , stress component; ϵ_{ij} , deformation rate component; σ_0 , ϵ_0 , stress and deformation amplitudes; φ , phase shift between deformation and stress; T_1 , relaxation time; T_2 , retardation time; θ , natural time; $Re = \rho v d / \eta$, Reynolds number; $We = \theta v / d$, Weissenberg number; Re^* , We^* , critical parameter values; $G'(\omega)$, elastic modulus; $G''(\omega)$, viscous loss modulus; $t_F(\omega)$, relaxation time spectrum.

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